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ENLISTED ADVANCEMENT OPTIMIZATION: A MULTIGOAL PROBLEM.(U)
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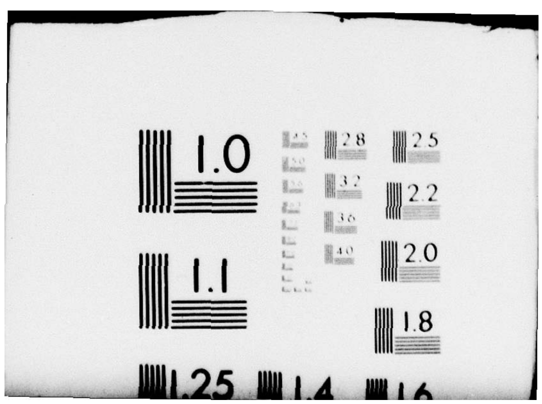
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ENLISTED ADVANCEMENT OPTIMIZATION: A MULTIGOAL PROBLEM.

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In planning promotions for the Navy's enlisted petty officer personnel, it is rarely possible to satisfy all goals simultaneously. These goals include (1) enlisted manpower requirements by grade and occupation, (2) personnel ceilings, (3) "minimization" of shortages and surpluses, (4) equal promotion opportunities among skill categories, and (5) satisfactory requirements for experience. A goal programming formulation is used to explore the structure of the problem and explictly identify possible tradeoffs among conflicting goals.

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FOREWORD

This research and development was conducted in support of reimbursable project POKV307, FAST Model Development, under the sponsorship of OP-16D (Decision Support Systems). The purpose of this project is to develop a comprehensive model of the enlisted personnel system using a combination of simulation and optimization techniques. This report presents an optimization "submodel" that will be incorporated into the FAST system.

DONALD F. PARKER
Commanding Officer

SUMMARY

Problem

Although control of petty officer promotions or advancements is the Navy's single most effective means for short-term management of the enlisted force, the advancement system must operate under a variety of requirements and constraints. In addition, the Navy may influence advancements by emphasizing particular occupational specialities or ratings required for fleet readiness. Currently, there are no quantitative techniques available that simultaneously consider both the systemic constraints and policy variables involved in enlisted advancement planning.

Objective

The purpose of this work was to investigate various aspects of the petty officer advancement system to develop a mathematical model for determining advancement. The output of the model is a distribution of advancements over ratings and pay grades that is "optimal" for the scenario under which the model is operating.

Model Formulation

Due to the multiple and conflicting objectives encountered in developing a plan for enlisted advancements, a linear goal programming approach was adopted. The model was formulated using constraints on manpower requirements, testtakers, waivers, tokens, and apportionment. A penalty is placed on the deviations between the goals and the computed values, and the advancements are distributed so that the penalties incurred are minimized.

Conclusions

Initial efforts of running the model have proven successful, in that desirable advancement distributions were achieved, subject to the ordering of the goals used. These results indicate that the model can be effective for dealing with the multigoal advancement problem.

Recommendations

This model should be considered for incorporation into the Force Structure Simulation Model (FAST), which supports the Navy's enlisted personnel system. The model could also be adapted for use in OPNAV's Advancement Plans Section, Enlisted Programs Implementation Branch (OP-135F).

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INTRODUCTION

Problem

The Navy's military system is a base-entry, nearly closed manpower system. In such a system, control of certain personnel flows is necessary to achieve management objectives. Although control of enlisted petty officer (E-4 through E-9) promotions or advancements is the Navy's single most effective means for short-term management of the enlisted force, the advancement system must operate under a variety of requirements and constraints. This includes upper and lower bounds on the number of advancements, minimum experience requirements, test scores obtained, and recommendation from higher authority. In addition, the Navy may influence advancements by emphasizing particular occupational specialties or ratings needed for fleet readiness. Where authorized strength levels are less than manpower requirements by rating, the amount of deviation from those requirements is subject to discretionary control.

Currently, quantitative techniques that can simultaneously consider the above listed systemic constraints and policy variables involved in petty officer advancement planning are not available. Without this capability, it is extremely difficult to determine coherent advancement policies in a changing environment.

Background

The U. S. Navy's enlisted personnel force can be characterized as a structure consisting of about 95 ratings (skill categories), each of which may exist at up to nine pay grades (skill levels). The grade structure consists of two distinct components: (1) the bottom three pay grades (E-1 through E-3), which are comprised largely of "nonrated" personnel (i.e., apprentices) who are undergoing training, and (2) the top six pay grades (E-4 through E-9), which are comprised of "rated" personnel who have undergone training. These rated personnel are called "petty officers," and their advancement, unlike that of nonrated personnel, is centrally controlled by personnel planners.

A "force structure" is an array wherein personnel in a given pay grade and rating are distributed by their length-of-service (LOS). Conventionally, there are 31 LOS cells, corresponding to years of active service. The first cell contains personnel with less than 1 complete year's service; the second, personnel with at least 1 year's service but less than 2; and so on, until the 31st cell, which contains personnel with 30 or more years of service.

The state of the enlisted personnel system at any point in time is the result of a large number of flows. Structural flows can be represented by movement between pay grades (e.g., demotions) or among ratings (e.g., laterals); and temporal flows, by movement between adjacent LOS cells (i.e., "aging"). In addition, there are flows into (e.g., recruits) and out of (e.g., attrites) the system. These flows have the effect of creating and filling vacancies in the various ratings and pay grades. For purposes of this report, vacancies for some time period are defined as the desired end period personnel inventory minus the net inventory for a particular rating and pay grade (the combination of which is called a "rate"). The net inventory is the personnel force at the beginning of a period less the net losses occurring during the period. All gains and losses, including advancements that are not centrally controlled such as automatic advancements, or advancements made under the Command Advancement Program (CAP), are counted in the net losses. Examined advancements are not counted.

Advancements required at a rate are computed by summing the vacancies at the particular pay grade being considered and at all higher pay grades. This method of advancement computation, which is known as "carry-down," has the effect of making advancements required at a specific pay grade independent of those made at higher pay grades. If too few people are advanced into the higher pay grades, excesses will appear in the inventories of the lower pay grades, assuming that there are sufficient advancement resources at those lower grades. Similarly, if too many people are advanced into the higher pay grades, there will be shortfalls in the inventory of the lower pay grades. The idea behind the "carry-down" procedure is to satisfy the total petty officer requirements of a rating, even when it is not possible to satisfy the requirements at a single pay grade.

While it is rarely a problem to advance the required number of petty officers for a pay grade at the TOTAL NAVY level, it may be undesirable or impossible to do so at the rating level. There are a variety of reasons for this:

1. Before individuals are considered eligible for advancement, they must, among other things, take and pass the advancement examination for the next higher pay grade. A person who has done this is called a "testtaker" or a promotion resource. There may not be enough testtakers at the preceding pay grade in particular ratings.

2. The desired end strength for a particular rating and pay grade (rate) is the number of people that has been determined to be an appropriate allocation of the TOTAL NAVY manpower authorization for that rating and pay grade at the end of the fiscal year. Since the current inventory is not considered a constraint in determining the desired end strength, there could be a large disparity between begin inventory and desired end strength, with no feasible means of reaching the desired end strength in the short run.

3. The carry-down methodology may result in "negative" advancements required for a rating at a particular pay grade.

4. For the general morale of the enlisted force, and to ensure that there is some advancement flow in every rate, the Navy supports a "token" advancement policy. Token advancements represent a lower bound on advancements, and may exceed the advancements required.

The factors that must be considered in determining an acceptable advancement strategy include experience requirements, allowable deviations from desired end strength, and systemic constraints inherent in the personnel system. These will be discussed in greater detail below.

Although both time-in-service (TIS) and time-in-grade (TIG) requirements are important considerations in advancement planning, only the TIS requirements will be considered here. This requirement assures that personnel meet minimum experience levels before they can be considered for advancement. It effectively divides the LOS distribution of a pay grade population into two zones--"waiver" and "preferred." The waiver zone includes personnel who do not meet the Navy-wide minimum TIS requirements but may, nevertheless, be considered as resources for advancement. To satisfy petty officer requirements for experience, attempts are made to restrict the percentage of advancements from this zone. The "preferred" zone includes personnel who meet the Navy's experience requirements. There is no restriction on the percentage of advancements that may come from this zone.

For reasons discussed above, it is frequently difficult to meet the desired end strength of every rating and the TOTAL NAVY. After advancements have been made in

every rating, the sum of those advancements over ratings in each pay grade is likely to be less than or greater than the number of personnel authorized at the TOTAL NAVY level. When this occurs, the advancement planner may attempt to iterate to a solution that satisfies the TOTAL NAVY authorization, and is within the bounds of a maximum and minimum end strength by rating. (It should be noted that the maximum and minimum end strength constraints may or may not apply when computing the rating advancements required.) This procedure, which is called "apportionment," is, indirectly, a reflection of the dual purpose of each petty officer position--it encompasses rating-specific technical skills and the military authority associated with "fighting the ship." The military responsibilities inherent in each petty officer grade, which constitute the foundation for military organization and discipline, may be independent of any technical responsibilities. As a result of this dual purpose, the requirement for TOTAL NAVY petty officers at any grade is independent of the requirement for specific technical skills.

Therefore, when TOTAL NAVY end strength for a pay grade cannot be met using the desired skill mix, military requirements must be met, even if the configuration of skills is slightly altered.

Objective

The purpose of this report is to describe a model that encompasses the systemic and policy constraints inherent in the advancement system. Once implemented, such a model would allow enlisted advancement planners to determine how planned or hypothetical changes would effect the advancement system.

MODEL FORMULATION

Due to the multiple and conflicting objectives encountered in developing a plan for enlisted advancements, a linear goal programming approach was adopted. Goal programming differs from conventional linear programming (LP) in that the objective function is a weighted summation of the deviations from the goals. This technique has the effect of simultaneously combining several criteria into one objective function.

The original formulation of the problem involved solving for six pay grades (E-4 through E-9), all ratings, and all LOS cells simultaneously. Unfortunately, this formulation would have resulted in over 20,000 row and bound constraints, and would have been too large a problem to be solved expediently using currently available LP software packages. To reduce the size of the problem, the detailed intervals in the LOS dimension were replaced by two zones, thus reducing the number of row and bound constraints to approximately 4400. The output of this model would be the number of advancements by zone, rather than by cell. These advancements could then be distributed into LOS cells (e.g., making advancements proportional to the LOS distribution of testtakers).

The problem was further reduced by running the model for one pay grade at a time. This procedure, together with the two-zone approach, reduces the number of row and bound constraints to approximately 740. Since the time taken to solve LP problems by computer tends to increase exponentially with the number of rows, it is both time and cost-effective to run six 740-row problems rather than one 4400-row problem.

Since the formulation being discussed in this report is for a single pay grade, the notations being used will not include subscripts to indicate relative pay grade, and "rating" will be used to refer to a rating at a specific pay grade.

Model Structure

To run effectively, the model needs a large amount of data (see Table 1). The data determines the status of the personnel system at the time advancements are to be made. The current source of this data is a force structure simulation model known as FAST (Boller, Lehto, Offir, & Silverman, 1978). These data are used almost exclusively as constants for the right-hand sides of the linear goal programming constraints. Interfacing the six separate pay grade problems is relatively easy. Due to carry-down, required advancements at a pay grade are independent of advancements made at other pay grades. The only adjustments that are necessary are to reduce the net inventory in order to reflect the individuals advancing to the next higher grade. Also, it will be necessary to adjust the maximum and minimum end strengths used to provide the slack necessary to achieve multiple goals. This information is available if the six goal programs (GPs) are solved using the "top down" approach (i.e., when the problem is solved for E-9 first, then E-8 based on the optimal solution for E-9, etc.).

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Table 1

Structural Data

Name	Description
a_i	Net inventory before advancements for rating i
b_i	Testtakers in the waiver (early) zone for rating i
c_i	Testtakers in the preferred zone for rating i
d_i	Desired end strength for rating i
e_i	Advancements required for rating i
f_i	Historical percentage of advancements occurring in the waiver zone of rating i
g	Advancements required for TOTAL NAVY ($\sum_i e_i$ if $\sum_i d_i$ = TOTAL NAVY authorization)
h_i	Vacancies for rating i ($h_i = d_i - a_i$)

The six separate GP solutions are no longer optimal in the same sense that the one larger GP solution would be. When the pay grades are optimized separately in the "top down" approach, it implies that it is more important to satisfy requirements at the higher pay grades than at the lower pay grades--in a sequential fashion. For example, if there were 10 advancements required for a particular rating at E-6 and 10 testtakers available, all 10 testtakers would be advanced to E-6--barring other restrictions. This could happen even if the 10 testtakers were the entire inventory for the rating at E-5, and there were no available testtakers at E-4 eligible for advancement to E-5, thus resulting in an E-5 inventory of <u>no</u> personnel. It should be noted, however, that this is a rather extreme example. Since the single pay grade GP takes no cognizance of inventories and testtakers in pay grades not within its scope, undesirable results may be produced at lower pay grades in very extreme cases. This condition might correspond to the results of the single large GP solution, if the penalties associated with the goal variables at succeedingly higher pay grades are an order of magnitude larger than the penalties at the lower pay grades.

Model Variables

The model variables are listed in Table 2 and described below.

1. <u>Goal Variables</u>. In a goal programming formulation, two slack goal variables are placed in each of the goal-oriented rows to measure the positive and negative deviations from the goals. The goal variables are associated with a specific penalty coefficient. Its value is the penalty incurred for each unit deviation from the goal. These coefficients are also listed in Table 2. (See the appendix for a strategy on choosing appropriate penalty coefficients.)

Table 2

Model Variables

Name	Definition
	Goal Variables[a]
k_i^-	Negative deviation from waiver policy goal for rating i. This is the number of people by which waiver advancements fall short of the policy goal.
k_i^+	Positive deviation from waiver policy goal for rating i. This is the number of people by which waiver advancements exceed the policy goal.
$l_{1_i}^-$	Negative deviation from requirements goal for rating i. This is the number of people by which the first penalty interval falls short of desired end strength.
$l_{2_i}^-$	Negative deviation from requirements goal for rating i. This is the number of people by which the second penalty interval falls short of the first penalty interval.
$l_{3_i}^-$	Negative deviation from requirements goal for rating i. This is the number of people by which the third penalty interval falls short of the second penalty interval.
$l_{1_i}^+$	Positive deviation from requirements goal for rating i. This is the number of people by which the first penalty interval exceeds the desired end strength.
$l_{2_i}^+$	Positive deviation from requirements goal for rating i. This is the number of people by which the second penalty interval exceeds the first penalty interval.
$l_{3_i}^+$	Positive deviation from requirements goal for rating i. This is the number of people by which the third penalty interval exceeds the second penalty interval.
m_i^-	Negative deviation from maximum end strength goal for rating i. This is the number of people by which the inventory after advancements falls short of the maximum end strength.
m_i^+	Positive deviation from maximum end strength goal for rating i. This is the number of people by which the inventory after advancements exceeds the maximum end strength.
n_i^-	Negative deviation from minimum end strength goal for rating i. This is the number of people by which the inventory after advancements falls short of the minimum end strength.

[a]All the above goal variables are considered to be nonnegative.

Table 2 (Continued)

Name	Definition
	Goal Variables[a] (Continued)
n_i^+	Positive deviation from minimum end strength goal for rating i. This is the number of people by which the inventory after advancements exceeds the minimum end strength.
p_i^-	Negative deviation from distribution goal for rating i. This is the number of people by which the waiver zone advancements fall short of the desired distribution ratio.
p_i^+	Positive deviation from distribution goal for rating i. This is the number of people by which the waiver zone advancements exceed the desired distribution ratio.
q^-	Negative deviation from waiver policy goal TOTAL NAVY. This is the number of people by which waiver advancements TOTAL NAVY falls short of the policy goal.
q^+	Positive deviation from waiver policy goal TOTAL NAVY. This is the number of people by which waiver advancements TOTAL NAVY exceed the policy goal.
r^-	Negative deviation from requirements goal TOTAL NAVY. This is the number of people by which end strength TOTAL NAVY falls short of the desired total.
r^+	Positive deviation for required advancements TOTAL NAVY. This is the number of people by which end strength TOTAL NAVY exceeds the desired total.
	Penalty Coefficients
δ_i	Penalty incurred for exceeding the waiver policy for rating i by one unit. The waiver policy has been exceeded if k_i^+ is positive.
ε_i	Penalty incurred for falling short of the advancements required for rating i by one unit. The advancements required have fallen short if $l_{1_i}^-$ is positive. (First penalty interval)
ζ_i	Penalty incurred for falling short of the advancements required for rating i by one unit. The advancements required have fallen short if $l_{2_i}^-$ is positive. (Second penalty interval)

[a]All the above goal variables are considered to be nonnegative.

Table 2 (Continued)

Name	Definition
	Penalty Coefficients (Continued)
η_i	Penalty incurred for falling short of the advancements required for rating i by one unit. The advancements required have fallen short if $l_{3_i}^-$ is positive (Third penalty interval)
θ_i	Penalty incurred for exceeding the advancements required for rating i by one unit. The advancements required have been exceeded if $l_{1_i}^+$ is positive (First penalty interval)
κ_i	Penalty incurred for exceeding the advancements required for rating i by one unit. The advancements required have been exceeded if $l_{2_i}^+$ is positive. (Second penalty interval)
μ_i	Penalty incurred for exceeding the advancements required for rating i by one unit. The advancements required have been exceeded if $l_{3_i}^+$ is positive. (Third penalty interval)
λ_i	Penalty incurred for exceeding maximum end strength for rating i by one unit. Maximum end strength has been exceeded if m_i^+ is positive.
ξ_i	Penalty incurred for falling short of minimum end strength for rating i by one unit. Minimum end strength has not been met if n_i^- is positive.
π_i	Penalty incurred for falling short of the desired distribution ratio for rating i by one unit. The distribution ratio has fallen short of the desired ratio if p_i^- is positive.
ρ_i	Penalty incurred for exceeding the desired distribution ratio for rating i by one unit. The distribution ratio has been exceeded if p_i^+ is positive.
T	Penalty incurred for exceeding the TOTAL NAVY waiver policy by one unit. The TOTAL NAVY waiver has been exceeded if q^+ is positive.
ϕ	Penalty incurred for falling short of the advancements required TOTAL NAVY by one unit. The advancements required have fallen short if r^- is positive.
ψ	Penalty incurred for exceeding the advancements required TOTAL NAVY by one unit. The advancements required have been exceeded if r^+ is positive.

Table 2 (Continued)

Name	Definition
	Decision Variables
x_i	Advancements to be made in the waiver zone of rating i.
y_i	Advancements to be made in the preferred zone of rating i.
	Policy Variables
s_i	Maximum desired percentage of advancements to occur in the waiver zone of rating i.
t	Maximum desired percentage of advancements to occur in the waiver zone TOTAL NAVY.
u_i	Minimum advancements (token policy) for rating i.
v_i	Maximum end strenth for rating i.
w_i	Minimum end strength for rating i.

2. Decision Variables. These are the model outputs. They represent an allocation of advancements that minimizes the penalties incurred.

3. Policy Variables. These variables represent policies that are controlled by the Navy, and may therefore be varied to test "what if" schemes. It should be noted that one of the more important policies--establishing the LOS limits for the advancement zones--is not defined here. This is because these zones are implicitly included in some of the previously defined variables/constants, particularly the available testtakers, and percentage restrictions in the zones.

Constraints

The six types of constraints involved in the multigoal advancement problem are discussed below.

1. Requirements Constraint. The requirements constraint dictates the number of advancements necessary to meet the desired end strength (d_i). This will be equal to the number of advancements required at a particular rating (e_i). The constraint is designed to give the goal programming model some "incentive" to come as close as possible to the desired end strength. Without this constraint, end strength at maximum or minimum levels would be interpreted as having the same benefits as at the desired level.

This constraint possesses great flexibility in that it can, to a large extent, control the apportionment process. With this constraint, penalties for deviation from desired end strength may be set so that "noncritical" ratings will be most likely to incur deviations from end strength (either positive or negative, depending upon how penalties are set), while "critical" ratings will tend to meet end strength whenever possible (due to penalties being set higher). In addition, the penalties for violation of this constraint are set up in steps, so that higher unit penalties may be incurred for successively larger deviations from end strength goals. This allows for a tolerance zone in which penalties may be minimal or nonexistent, with discontinuously larger penalties associated with distance from this zone. For example, the penalty may be set to one unit for a deviation from desired end strength of one person, to four units for deviation of two and three people, and to ten units for deviations in excess of three people. The penalties and the regions may be set differently by rating for positive and negative deviations.

The requirements constraint exists for both ratings and TOTAL NAVY. The TOTAL NAVY version of this constraint is similar to that of the ratings, except that there are only single goal variables (r^+, r^-) for positive and negative deviations, as opposed to the multiple goal variables ($l^-_{1_i}$ $l^-_{2_i}$, $l^-_{3_i}$, $l^+_{1_i}$, $l^+_{2_i}$, $l^+_{3_i}$) used for ratings. The TOTAL NAVY goal variables would normally have much higher penalties assigned to them than the rating goals, due to the importance of satisfying TOTAL NAVY end strength. In most planning scenarios, TOTAL NAVY goals would be considered of prime importance.

So,

$$e_i = x_i + y_i + l^-_{1_i} + l^-_{2_i} + l^-_{3_i} - l^+_{1_i} - l^+_{2_i} - l^+_{3_i} \qquad \text{(for all ratings i)}$$

$$0 \leq l^-_{1_i} \leq A_i$$

$$0 \leq l_{2_i}^- \leq B_i$$

$$0 \leq l_{3_i}^-$$

$$0 \leq l_{1_i}^+ \leq C_i$$

$$0 \leq l_{2_i}^+ \leq D_i$$

$$0 \leq l_{3_i}^+$$

where A_i, B_i, C_i, and D_i define the regions for rating i for the different penalties for missing desired end strength, and

$$g = \sum x_i + \sum y_i + r^- - r^+ \qquad \text{(for TOTAL NAVY)}$$

2. Maximum End Strength/Minimum End Strength Constraints. The purpose of these constraints, which are very similar to each other, is to ensure that the apportionment segment of the model will not cause the end strength for a rate to exceed or fall short of predetermined bounds. The equations have two forms: one at the E-9 level, where there are no adjustments to be made to compensate for carry-down effects at higher pay grades, and the other for the E-4 through E-8 levels. Adjustments are made at these pay grades (a) to remove people included in the net inventory who have already advanced into the next higher pay grade, and (b) to adjust the maximum and minimum end strengths to reflect excesses or shortages in the inventory due to inability to meet the desired number of advancements at the next higher pay grade. The maximum and minimum end strengths, and the penalties for failing to meet them may be set independently for every rate.

So,

$$x_i + y_i + m_i^- - m_i^+ = v_i - a_i \qquad \text{(max end strength E-9)}$$

$$x_i + y_i + m_i^- - m_i^+ = v_i - a_i + x_i^* + y_i^* + l_{1_i}^{-*} + l_{2_i}^{-*} + l_{3_i}^{-*}$$

$$- l_{1_i}^{+*} - l_{2_i}^{+*} - l_{3_i}^{+*} = v_i - a_i + e_i^* \qquad \text{(max end strength E-4 through E-8)}$$

$$x_i + y_i + n_i^- - n_i^+ = w_i - a_i \qquad \text{(min end strength E-9)}$$

$$x_i + y_i + n_i^- - n_i^+ = w_i - a_i + x_i^* + y_i^* + l_{1_i}^{-*} + l_{2_i}^{-*} + l_{3_i}^{-*}$$

$$- l_{1_i}^{+*} - l_{2_i}^{+*} - {}^{+*}_{3_i}$$

$$= w_i - a_i + e_i^* \qquad \text{(min end strength E-4 through E-8)}$$

The above set of equations may be used if the maximum and minimum end strengths are used only to constrain the apportionment process. If the computation of the advancements required is also to be constrained by maximum and minimum end strengths, the following equations may be used for the E-4 through E-8 pay grades.

$$x_i + y_i + m_i^- - m_i^+ = v_i - a_i + x_i^* + y_i^* \qquad \text{(max end E-4 through E-8)}$$

$$x_i + y_i + n_i^- - n_i^+ = w_i - a_i + x_i^* + y_i^* \qquad \text{(min end E-4 through E-8)}$$

An (*) denotes that the value of the variable is for the next higher pay grade. The values for these variables have been determined in the previous pay grade run, and are necessary inputs for the current stage of the model.

3. Token Constraints. The token constraint ensures that the advancements made are equal to or exceed the token policy. In the current formulation of the model, the token constraint is one of the two "hard" constraints; that is, it is a constraint without goal/slack variables. It must therefore be satisfied to produce a feasible solution. This constraint will not cause the model to produce an infeasible solution if the token advancements (u_i) are set equal to some percentage of the testtakers. This ensures that the token policy can always be met. This constraint takes the form,

$$x_i + y_i \geq u_i. \qquad \text{(For all ratings i)}$$

4. Waiver Zone Constraint. The waiver zone constraint is used to limit the number of advancements made in the waiver zone (expressed as a percentage of total advancements made in the rating or TOTAL NAVY). While the waiver policy exists only at the TOTAL NAVY level, it may be desirable to control waivers at the rating level. The user is therefore free to set differing waiver policies from rating to rating, including the capability to "turn off" the waiver policy for particular ratings. Eliminating the waiver policy for a rating indicates that the user is not particularly concerned about the percentage of early zone advancements for that rating.

In addition to being able to change the constraint from rating to rating, it is possible to vary the penalty for not adhering to the "policy" from rating to rating. This gives one the ability to decrease penalties for "critical" ratings (ratings in which it is deemed necessary for end strength to be reached if at all possible) while maintaining

higher penalties for "noncritical" ratings, where shortfalls would not have as detrimental an effect on fleet readiness.

Although the waiver zone constraint exists at both the rating and the TOTAL NAVY level, the penalty imposed on the TOTAL NAVY constraint should generally be significantly larger than the rating penalties. This is because TOTAL NAVY policy is imposed by the Department of Defense and must be met, if at all possible, while the rating policies are user-imposed and can be sacrified to satisfy TOTAL NAVY needs.

Notationally,

$$0 = (1 - s_i)x_i - s_i y_i + k_i^- - k_i^+ \qquad \text{(for all ratings i)}$$

$$0 = (1 - t) \sum_i x_i - t \sum_i y_i + q^- - q^+ \qquad \text{(TOTAL NAVY)}$$

5. Distribution Constraint. The distribution constraint is used to simulate a systemic constraint inherent in the advancement system. Currently, the advancement planner is not empowered to determine the distribution of advancements over LOS (and hence the zones). The testtakers within a rating are ordered according to a point-system, and advancements for the rating must be chosen according to this predetermined order. Therefore, if the model determines that a rating should have 15 advancements in the waiver zone and 10 in the preferred zone, the information is useful only if the model is being used for policy testing or feasibility studies, as this "optimal" distribution is unlikely to correspond to the distribution over the waiver and preferred zones of the actual top 25 test passers. To make the model useful in a predictive or forecasting role, the expected ordering of testtakers is input into the model as a constraint (Milch, 1971, 1976). Hence, when the model is run with an appropriate penalty on this constraint, the "optimal" solution for a rating should be quite close to its actual distribution of testtakers.

This constraint exists for all ratings. The capability to vary the penalty for failure to meet this constraint exists by rating, although this would not seem to be appropriate in most scenarios (except perhaps in cases where the data being used to compute the ordering for a particular rating is of dubious quality, in which case the penalty for failing to meet the constraint should be lessened or eliminated).

$$0 = (1 - f_i)x_i - f_i y_i + p_i^- - p_i^+ \qquad \text{(for all ratings i)}$$

6. Testtaker Constraints. The testtaker constraint ensures that the number of advancements made in each of the two advancement zones (x_i, y_i) do not exceed the number of testtakers in that zone (b_i, c_i). This constraint, like the token constraint, contains no slack/goal variables and must be satisfied.

$$x_i \leq b_i$$

$$y_i \leq c_i \qquad \text{(for all ratings i)}$$

Objective Function

The "optimal" solution for a particular scenario is reached when the objective function is at a minimum. The objective function for this problem is the weighted summation of all the goal variables with the weights corresponding to the penalties associated with each of the goal variables:

$$\text{min:} \quad \sum_i \delta_i k_i^+ + \sum_i \epsilon_i l_{1_i}^- + \sum_i \zeta_i l_{2_i}^- + \sum_i \eta_i l_{3_i}^- + \sum_i \theta_i l_{1_i}^+ + \sum_i \kappa_i l_{2_i}^+$$

$$+ \sum_i \mu_i l_{3_i}^+ + \sum_i \lambda_i m_i^+ + \sum_i \xi_i n_i^- + \sum_i \pi_i p_i^- + \sum_i \rho_i p_i^+$$

$$+ \tau q^+ + \phi r^- + \psi r^+$$

CONCLUSIONS

Initial effects of running the model have proven successful, in that desirable advancement distributions were achieved subject to the ordering of priorities used. These results indicate that the model can be an effective tool in dealing with the multigoal advancement problem. Additional work will be done to determine appropriate penalties for a series of scenarios. This may involve the development of another model to employ an analytic approach for the computation of penalties. Also, the possibility of reformulating the problem in such a way as to take advantage of efficient network codes will be investigated (Thompson, 1978).

RECOMMENDATIONS

This model has obvious applications for the Force Structure Simulation Model (FAST), which supports the Navy's enlisted personnel planning system. Currently, the FAST model cannot handle the multigoal nature of the advancement problem explicitly. Although incorporating the proposed advancement optimization model would alleviate this deficiency, this step would involve considerable development in terms of penalty computation.

The model could also be adapted for use in authorizing actual (vice planned) advancements in OP-135F. This would involve adding additional constraints (budgeting) and changing the model's planning cycle from 1 year to 6 months. This application, however, would dramatically increase the problem size as well as require additional software development necessary for operational use.

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APPENDIX

CHOOSING APPROPRIATE PENALTIES FOR A PARTICULAR SCENARIO

CHOOSING APPROPRIATE PENALTIES FOR A PARTICULAR SCENARIO

Because of the structure of the linear goal programming formulation of the advancement problem, values may be assigned systematically to the various penalty variables to yield a desired result. One possible scenario is presented below.

Ordering of priorities:

1. No violation of maximum/minimum end strength by rating.
2. Achieve TOTAL NAVY end strength.
3. No violation of TOTAL NAVY waiver policy.
4. Achieve rating end strength.
5. Achieve preferred rating distribution of advancements.
6. No violation of rating waiver policy.

The priorities have been listed in decending order, i.e., 1 is our most critical priority or goal, while 6 is least critical. Here, we assume that the relative importance of satisfying the goals is cumulative and, therefore, it is more desirable to satisfy goal 1 than goals 2 through 6 combined. In other words, it is more important to avoid violating the maximum or minimum rating end strength than it is to satisfy any of the five other constraints. Similarly, the satisfaction of goal 2 carries with it a higher "reward" or benefit than the satisfaction of goals 3 through 6 combined. Based on this understanding, numerical values may be assigned to the various penalties.

Rating Waiver Policy

Since it is desirable to allow for some means of expressing rating "utility" within this constraint, a range of penalties of from one to three units might be arbitrarily assigned to this goal. Thus, for the ratings in which satisfaction of the waiver policy constraint is considered noncritical, a penalty of one unit would be incurred for the advancement of each person in excess of the rating waiver policy. Similarly, a penalty of three units would be incurred in the "critical" ratings for each advancement in the waiver zone exceeding waiver policy.

Rating Distribution Constraint

Since, for purposes, of this scenario, this constraint has greater importance than the rating waiver constraint, it must be assigned a penalty higher than that constraint. Therefore, the penalty assigned for violation of the distribution goal will be a range of from four to six units for each advancement by which the rating distribution goal is violated. (As in the previous constraint, the range of penalties is provided to allow the user to distinguish between "critical" and "noncritical" ratings within this constraint.) This assures that, within a rating, the optimal solution will try to satisfy the rating distribution constraint rather than the rating waiver constraint, <u>if</u> all other constraints are equally well met under either potential solution. (This is because the optimal solution will minimize the penalties incurred, and a penalty of one to three units is more desirable than one of four to six units.)

Rating End Strength Constraint

Because this constraint has three sets of goal variables associated with it, each of them must be considered individually.

In this scenario, it was assumed that the three penalty intervals, like the goals discussed above, are cumulative in importance. For example, even if moving an excess advancement from the first penalty interval of one rating to the second penalty interval of another rating (which already has enough excess advancements to fill its first penalty interval) would improve both ratings waiver and distribution goals, it would be an undesirable effect. The ordering of priorities may be reconsidered as follows:

1. No violation of maximum/minimum end strength by rating.
2. Achieve TOTAL NAVY end strength.
3. No violation of TOTAL NAVY waiver policy.
4. No occurrences of third interval penalties.
5. No occurrences of second interval penalties.
6. No occurrences of first interval penalties.
7. Achieve preferred rating distribution of advancements.
8. No violation of rating waiver policy.

First Penalty Interval

The initial step is to determine the maximal "benefit" that may be derived from satisfying the lesser constraints in the scenario. In this case, the shift of an advancement from one rating to another could, under some circumstances, result in both ratings coming closer to satisfying their distribution and waiver constraints. This would result in a maximal "benefit" of 3 + 3 (from waiver constraint) + 6 + 6 (from distribution constraint) for a total of 18. For reasons discussed above, it is desirable to have an interval, in this case, say, 19 to 23. This ensures that an advancement would not be moved from one rating to another, solely to incur a lesser penalty for the violation of the rating waiver and distribution constraints.

Second Penalty Interval

The maximal "benefit" that may be derived from removing/adding an advancement falling within the second penalty interval of the requirements constraint is, once again, the satisfaction of the lesser constraints. Maximal benefit is therefore 3 + 3 + 5 + 5 + 23 = 39 (for the case where an advancement is moved out of a rating's first penalty interval and improves the waiver and distribution goals of both ratings). A range of 40 to 44 will therefore be used for the penalties of the second interval.

Third Penalty Interval

The computation is almost identical to the second interval, with maximal benefit being 3 + 3 + 5 + 5 + 44 = 60. A range of 61 to 65 should therefore work out satisfactorily for the penalties assigned to the third interval.

TOTAL NAVY Waiver Policy

Maximal benefit for violation of this constraint is derived in the case where an advancement is moved from one rating to another and both ratings improve upon their waiver constraints (3 + 3), their distribution constraints (6 + 6), and, in addition, decrease their third interval requirement penalties (65 + 65), for a total of 148. A value of 149 may therefore be chosen for this penalty variable.

TOTAL NAVY End Strength Constraint

Maximal benefit derived from violation of this constraint occurs in the instance where an advancement is added or removed from a rating, and not apportioned elsewhere.

Computationally, this benefit would be 3 (rating waiver) + 6 (rating distribution) + 65 (rating end strength) + 149 (TOTAL NAVY waiver) = 220. A penalty of 221 may therefore be assigned.

Maximum/Minimum End Strength Constraint

Violation of this constraint could result in decreased penalties for one rating in the areas of rating waiver (3), rating distribution (6), TOTAL NAVY waiver (149), and TOTAL NAVY end strength (221), for a total of 379, allowing an assignment of 380 for this penalty.

This completes the process of determining "reasonable" penalty variables for this scenario. The same procedure may be used to compute penalty variables for the constraints under any ordering of priorites.

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